numerical problem described earlier.[1] Except for those points the over-all accuracy for these data is believed to be about 5%.

The density profile at $x/D = 5$ (Fig. 7) is similar in the expansion region as discussed before except that the relative minimum in density at the edge of the inviscid wake is terminated by the strong wake shock which compresses and turns the expanding flow near the axis. The higher density on the wake axis, relative to the profile at $x/D = 1$, is related mainly to the higher pressure in this region compared to the base region. No experimental data has been found or flowfield calculations made for comparison purposes with these data.

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Flutter of Buckled Plates at Zero Dynamic Pressure

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In Ref. 1–5 the anomalous flutter behavior of certain buckled plates has been discussed. As is often the case, the anomaly appears more glaringly in the theoretical results than in the experimental data. In particular for certain combinations of plate length/width ratio and applied in-plane compressive loadings, the conventional theory predicts the plate will become unstable at zero dynamic pressure. The necessary combinations are those for which two modes have identical and zero natural frequencies at some level of inplane load. In point of fact, the instability is a static one, i.e., the plate buckles rather than flutters. Nevertheless, the conventional flutter theory, defined as a linear model whose structural and aerodynamic damping go to zero as the frequency of oscillation goes to zero, predicts that for all finite dynamic pressure, flutter (an exponentially diverging oscillation) will occur. No such behavior is observed experimentally.

Shore,[1] in an effort to resolve the differences between theory and experiment, has included a form of structural damping in the theory which remains finite even when the plate is motionless. Such damping eliminates the zero dynamic pressure flutter; however, the present writer finds the physical significance of such damping somewhat obscure.

In this Note we investigate, through a nonlinear analysis, the actual flutter motion and the effect of initial plate imperfections. This will hopefully improve the physical understanding of this problem and also provide a rational basis for design. We do not specifically study structural damping of any form; however, aerodynamic damping is included through the quasi-steady supersonic approximation. A future study might well investigate the implications of the present results with regard to the role of structural damping.

Received September 24, 1969; revision received November 21, 1969. This work was supported by NASA Grant NGR 31-001-146. The author would like to thank C. P. Shore and R. W. Hess of NASA Langley for helpful discussion.

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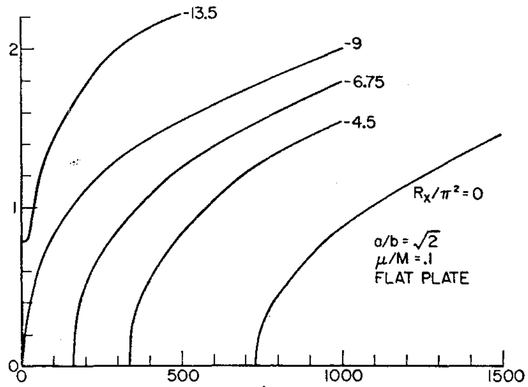

Fig. 1 Plate deflection vs dynamic pressure.

We choose for our study a simply supported plate of length/width ratio $a/b = 2^{1/2}$, and under a streamwise compressive load with no spanwise compressive load. Results will be presented in terms of the following nondimensional quantities (see Ref. 5–7): w/h = ratio of plate deflection/plate thickness, λ = nondimensional dynamic pressure, μ/M = nondimensional mass ratio, K = nondimensional frequency, R_x = nondimensional in-plane load, H/h = ratio of plate rise height/thickness.

In Fig. 1, we plot the peak deflection $(w/h)_p$ vs λ for several values of in-plane load. A typical value of mass ratio is used. For small in-plane load, the plate is flat and stable up to some value of dynamic pressure, after which flutter occurs. For $R_x/\pi^2 = -9$, "flutter" begins at $\lambda = 0$ and continues for all $\lambda > 0$. Note however that the plate amplitude is zero for $\lambda = 0$ and remains small for small λ. For $R_x/\pi^2 = -13.5$, the plate amplitude is finite at $\lambda = 0$. The plate is actually buckled for small λ and begins to flutter for $\lambda \gtrsim 50$. The frequency K is shown in Fig. 2, where the distinction between flutter and buckling is made somewhat clearer. Note in particular that for $R_x/\pi^2 = -9$, the frequency smoothly approaches zero as $\lambda \rightarrow 0$. Although not as relevant to our present concern, note the rapid variation of K for $R_x/\pi^2 = -13.5$ at small λ. This is a result of an "oil-canning" type oscillation whereby the plate moves from one buckled equilibrium position to another.

The first important observation is that while buckling or divergence occurs at $\lambda = 0$, the plate amplitude is zero. Hence any observable instability will be flutter at some finite amplitude and frequency.

Now let us turn to the effect of initial imperfections, i.e. any small deviation of the plate from perfect flatness. In Fig. 3 we give some of the results from Fig. 1 as well as additional results for a plate whose initial shape is parabolic in the streamwise direction with a maximum rise height H of one plate thickness. For small λ, the plate statically deforms under the aerodynamic load. For sufficiently large λ the plate flutters. A range of flutter amplitude is shown since

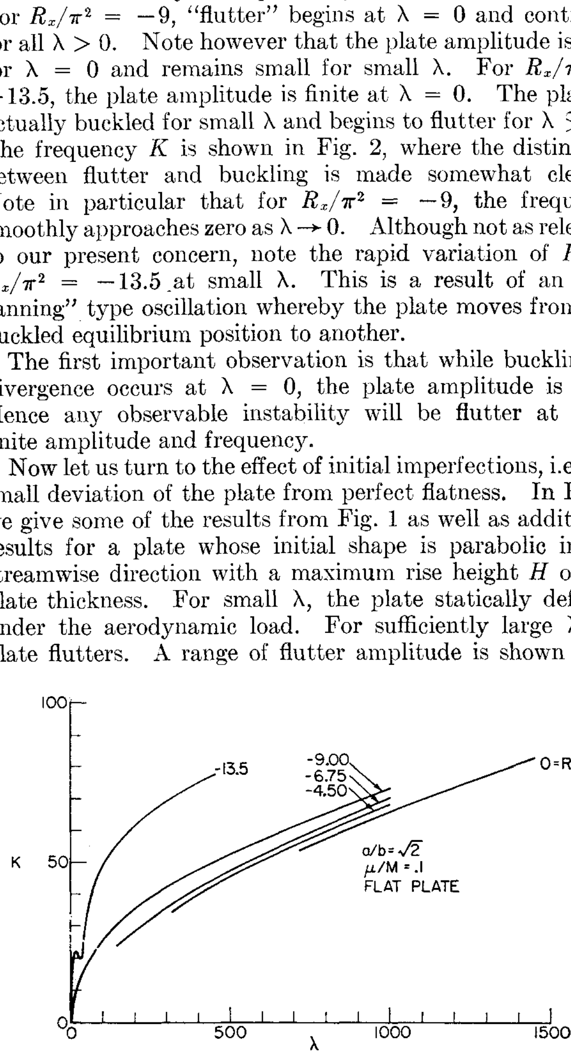

Fig. 2 Frequency vs dynamic pressure.

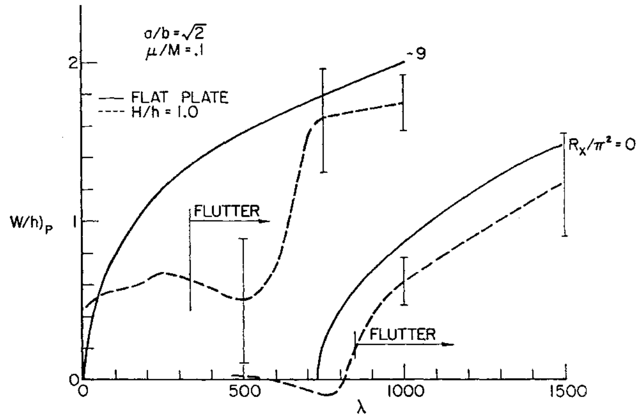

Fig. 3 Plate deflection vs dynamic pressure.

the flutter oscillation of an imperfect plate is no longer symmetrical about its shape. As may be seen the change from the flat plate result is very modest for no in-plane load $R_x/\pi^2 \equiv 0$; however, for $R_x/\pi^2 = -9$ the change is very substantial. Hence in Fig. 4 we plot the dynamic pressure at the onset of flutter, λ_f vs maximum rise height for $R_x/\pi^2 = -9$. Note the significant effect even for imperfections small compared to the plate thickness. Finally in Fig. 5 we plot λ_f vs R_x for a flat plate and an imperfect plate.

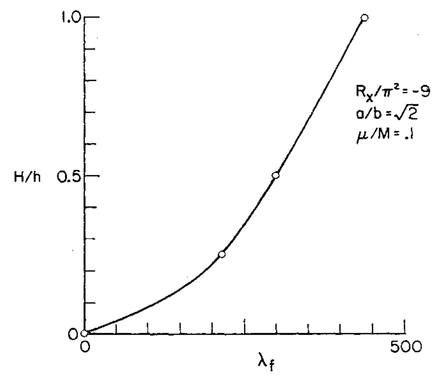

Fig. 4 Rise height vs flutter dynamic pressure.

From these results we make a second important observation. For an imperfect plate flutter will never begin at $\lambda \equiv 0$. However for nearly perfect plates it should be possible to approach this result.

Conclusions

Based upon the present results, and quite aside from any possible effect of structural damping, it is seen that any realistic, i.e., imperfect, plate will not flutter at zero dynamic pressure. The flutter dynamic pressure will depend upon the magnitude (and distribution) of the initial imperfections. Further the present results suggest the difficulty of experimentally establishing the flutter boundary under conditions at very low dynamic pressure. While the plate will flutter, the amplitude may be sufficiently small so that it is indistinguishable from plate response to flow noise. Finally, even for a perfect plate in the absence of flow noise, an aeroelastic instability only occurs under trivial conditions at zero dynamic pressure, i.e., zero plate amplitude.

The implications for design are obvious. A rational design may be based on either 1) maximum permissible plate deflection or stress or 2) the inclusion of realistic plate imperfections.

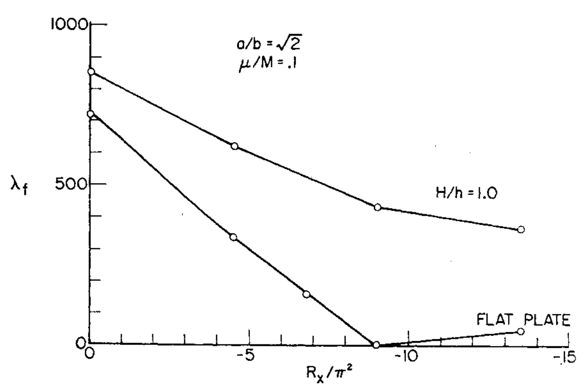

Fig. 5 Flutter dynamic pressure vs in-plane load.

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Stagnation Temperature and Molecular Weight Effects in Jet Interaction

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In a recent paper by Chrans and Collins,[1] the effects of injectant stagnation temperature and injectant molecular weight variations on the flowfield generated by secondary injection were determined experimentally. The purpose of this Note is to extend their work to consider the effect of these two variables on the interaction side force and the amplification factor. Relationships between the side force and penetration height (h) and between side force and the characteristic radius (R_o) of the blast wave theory are also developed.

Experiments were conducted in the Naval Postgraduate School supersonic wind tunnel at a primary Mach number of 1.92. The interaction side force was determined from the integration of the pressure measured from 47 pressure ports in the base of the wind tunnel. The geometric pattern of the pressure ports and the associated recording system are discussed in Refs. 2 and 3. The ratio of injectant stagnation pressure to freestream stagnation pressure ($P_{oj}/P_{o\infty}$) ranged from $\frac{2}{3}$ to 6, whereas the ratio of injectant stagnation tem-

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Received October 3, 1969; revision received November 24, 1969. This was supported by Naval Air Systems Command Contract A33-330/551/69.

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